

Increased Efficiency in the Second-Hand Tire Trade Provides Opportunity for Dengue Control

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Abstract

Dengue fever is increasing in geographical range, spread by invasion of its vector mosquitoes. The trade in second-hand tires has been implicated as a factor in this process as they act as mobile reservoirs of mosquito eggs and larvae. Regional transportation of tires can create linkages between rural areas with dengue to disease-free urban areas, potentially giving rise to outbreaks even in areas with strong local control measures. In this work we sought to model the dynamics of mosquito transportation via the tire trade, in particular to predict its role in causing unexpected dengue outbreaks through vertical transmission of the virus across generations of mosquitoes. We also aimed to identify strategies for regulating the trade in second-hand

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tires, improving disease control. We created a mathematical model which captures the dynamics of dengue between rural and urban areas, taking into account the movement, storage time of tires, and mosquito diapause. We simulate a series of scenarios. First a mosquito population is introduced to a dengue-free area via movement of tires, either as single or multiple events, increasing the likelihood of a dengue outbreak. An endemic state can be induced regardless of whether urban conditions for an outbreak are met, and an existing endemic state can be enhanced by vector input. Finally we assess the potential for regulation of tire processing as a means of reducing the transmission of dengue fever using a specific case study from Puerto Rico. Our work demonstrates the importance of the second-hand tire trade in modulating the spread of dengue fever across regions, in particular its role in introducing dengue to disease-free areas. We propose that regulation of tire storage and movement can play a crucial role in containing outbreaks and dengue spread.

Keywords:

Aedes; vertical transmission; diapause; reservoirs; transportation; mobility

Introduction

Dengue fever is among the most widespread vector-borne diseases, with approximately 2.5 billion people at risk and 50 million infections annually World Health Organization (2009). Dengue is endemic in over 100 tropical and subtropical countries Gubler (2002). It is also the fastest re-emerging dis-

ease C.Cook and I.Zumla (2008), imposing an economic burden alongside the impaired health of affected individuals. Two mosquito species are responsible for transmission of the virus via infective bites. The most common vector is *Aedes aegypti*, but the asian tiger mosquito *Aedes albopictus* is increasingly important due to a rapidly expanding global distribution encompassing most tropical regions Belli et al. (2015); Rezza (2012). There are four dengue virus serotypes García-Rivera and Rigau-Pérez (2006), and once an individual has been infected by one serotype they are permanently immune to that serotype but only temporarily immune to the others García-Rivera and Rigau-Pérez (2006); Esteva and Vargas (2003).

Second-hand tires are widely traded both locally and globally. In countries with *Ae. aegypti* mosquitoes, these often contain standing rain water and eggs Rezza (2012); Yee (2008a), providing excellent larval habitats and frequently infected with both species Alves Honório et al. (2006); Higa et al. (2010). Tires have been an important dispersal mechanism for both mosquitoes and dengue virus. *Ae. albopictus* originated in Asia but invaded the New World in the 1980s via imported used tires and bamboo plants Belli et al. (2015); Gubler (2002). It is now present in 20 countries in the Americas Belli et al. (2015). International trade in used tires and bamboo has also been implicated in the introduction of *Ae. albopictus* to Europe Medlock et al. (2012).

Ae. aegypti was eliminated in 1960 in almost all the New World. However, subsequent social and economic changes in the Americas have permitted the

rapid re-infestation of the vector and dengue virus throughout the region. From 1960 to 1990, the annual production of tires increased from 2 to 17 million Briseño-García et al. (1996), suggesting that the management of their movement, disposal and recycling could be an important modulator of dengue dispersal. For example, between 1981 and 1996, Cuba lacked any dengue transmission. Reintroduction has now occurred in some areas; the municipality of Santiago de Cuba was reinfested in 1992 by *Ae. aegypti* transported in tires Kourí et al. (1998).

Two processes play an important role in the transmission of *Aedes* and dengue fever via tires. The first is the diapause phase in the mosquito life cycle, enabling eggs to survive long periods of unfavorable conditions, including desiccation Thomas et al. (2012). Vertical or transovarial transmission of dengue also occurs, with infected females passing the virus to their eggs Esteva and Vargas (2000); Günther et al. (2007); Martins et al. (2012); Murillo et al. (2014). Emerging adults are therefore able to transmit the disease without first interacting with an infected host Gubler (1986); C.Cook and I.Zumla (2008), potentially causing outbreaks in dengue-free areas.

Here we assess the potential role of transportation of tires containing infected eggs in causing outbreaks in areas otherwise free of both vectors and dengue. Our mathematical model is based on two patches, representing a rural area with endemic dengue and an urban area which begins as dengue-free. We incorporate vertical transmission, diapause during transportation, and the efficiency of tire processing. Through this we generate scenarios

in which (a) mosquitoes arrive but with no dengue outbreak, (b) a dengue outbreak occurs, (c) an effective endemic state is created due to continuous influx of infected eggs, and (d) an existing endemic infection is enhanced through additional input of infected vectors. In order to assess the potential for management, we present a case study of implementing a management program to reduce tire processing times. Our work demonstrates that, if effectively regulated, a reduction in the time that tires are stored could aid in dengue control.

Methods

Our model aims to capture the dynamics of dengue fever in both humans and female mosquitoes. The total human population N remains constant throughout. The modeled populations are divided into rural and urban sections, with movement occurring only in the reservoirs containing eggs. The systems in the two patches differ mainly by the loss and gain of eggs from rural to urban areas. The total human populations in rural area N_R and urban area N_U are constant. Table 1 describes the population classes.

The susceptible human class has a per-capita birth rate η , and an identical per-capita death rate η , meaning that the overall population remains stable through time. Individuals in this class become infectious according to the bite rate of infected vectors α . The vectors become infected by biting infectious hosts with an identical contact rate α . The rate at which humans recover from infection, whereupon they become permanently immune, is γ .

Population	Description
S_R	Susceptible human in rural area.
I_R	Infectious human in rural area.
R_R	Recovered human in rural area.
S_U	Susceptible human in urban area.
I_U	Infectious human in urban area.
R_U	Recovered human in urban area.
M_{SR}	Susceptible mosquitoes in rural area.
M_{IR}	Infectious mosquitoes in rural area.
E_{SR}	Susceptible eggs in rural area.
E_{IR}	Infectious eggs in rural area.
M_{SU}	Susceptible mosquitoes in urban area.
M_{IU}	Infectious mosquitoes in urban area.
E_{SU}	Susceptible eggs in urban area.
E_{IU}	Infectious eggs in urban area.

Table 1: State variables.

In contrast, mosquitoes never recover from the disease. Our model only considers the fraction κ of mosquitoes that are female, as males do not transmit the disease. Mosquito populations increase through egg eclosion according to the development rate ω and individuals die with a rate ϵ . Female mosquitoes oviposit at a rate ϕ and the eggs have an intrinsic mortality rate π . If a female mosquito is already infected, a fraction ν of its oviposited eggs are infected (vertical transmission). The hatcheries have a carrying capacity C_a where $a \in \{r, u\}$, and r is rural and u urban area.

The number of tires transported from rural area to urban area per unit time is r and θ is the mean number of tires in the rural area. Hence rE_{IR}/θ is the rate of egg movement. During transportation a fraction χ of eggs survive. $\psi(\tau_s/\tau_d)$ is the fraction of eggs in the tires that were able to hatch as adults

before being killed by the recycling process of the tire. Thus it is a function of the storage time τ_s before tire processing and the egg development time τ_d . ψ should be a function such that when $\tau_s = 0$ then $\psi = 0$, while when τ_s is greater than the development time of the eggs τ_d , ψ should approach one. A flow diagram of the model system is shown in Figs 1 and 2. Table 2 shows a summary of parameters.

Parameter	Description
η	Per-capita birth and natural mortality rates in humans
γ	Per-capita recovery rate
α	Effective biting rate, per day
C_a	Carrying capacity of hatcheries, where $a \in \{r, u\}$, and r is rural and u urban area
ϕ	Number of eggs laid per day for every female mosquito
ϵ	Per-capita mortality rate of adult mosquitoes
π	Per-capita mortality rate of immature stage mosquitoes
ν	Proportion of eggs that are infected by vertical transmission
ω	Development rate of immature to mature stages
κ	Fraction of mosquitoes that are female
$\frac{r}{\theta}$	Per-capita egg transportation rate
χ	Fraction of eggs that survive the transportation
$\psi(\tau_s/\tau_d)$	Fraction of eggs in tires that were able to continue their development before tire processing
τ_s, τ_d	Tires storage time and egg development time

Table 2: Model parameters

Following the assumptions and parameters defined above, the system of differential equations that model the dynamics of dengue in human and mosquito populations in the rural area is given by:

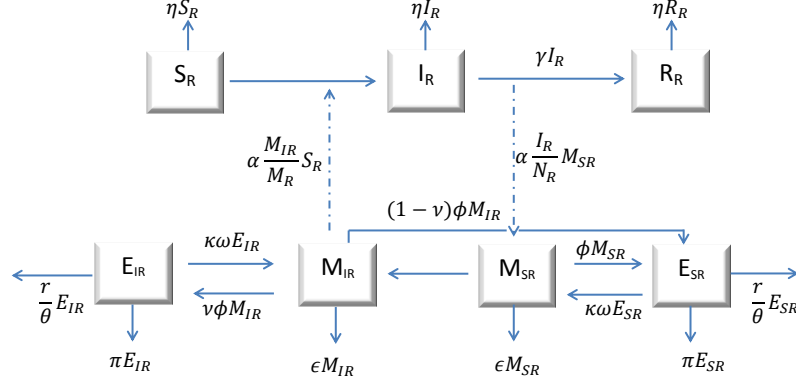


Figure 1: **Flowchart from rural dengue fever model.** Elements of the upper row refer to segments of the human population, whereas the lower row refers to adult mosquitoes (M) or their eggs (E). See Tables 1 and 2 for definitions of terms.

$$\begin{aligned}
\dot{S}_R &= \eta N_R - \alpha \frac{S_R}{N_R} M_{IR} - \eta S_R, \\
\dot{I}_R &= \alpha \frac{S_R}{N_R} M_{IR} - (\eta + \gamma) I_R, \\
\dot{R}_R &= \gamma I_R - \eta R_R, \\
\dot{M}_{SR} &= \kappa \omega E_{SR} - \alpha \frac{I_R}{N_R} M_{SR} - \epsilon M_{SR}, \\
\dot{M}_{IR} &= \kappa \omega E_{IR} + \alpha \frac{I_R}{N_R} M_{SR} - \epsilon M_{IR}, \\
\dot{E}_{SR} &= \phi M_{SR} \left(1 - \frac{E_R}{C_r}\right) + (1 - \nu) \phi M_{IR} \left(1 - \frac{E_R}{C_r}\right) - \left(\pi + \omega + \frac{r}{\theta}\right) E_{SR}, \\
\dot{E}_{IR} &= \nu \phi M_{IR} \left(1 - \frac{E_R}{C_r}\right) - \left(\pi + \omega + \frac{r}{\theta}\right) E_{IR}.
\end{aligned}$$

The differential equations that model the dynamics of dengue disease in human and mosquito populations in the urban area are given by:

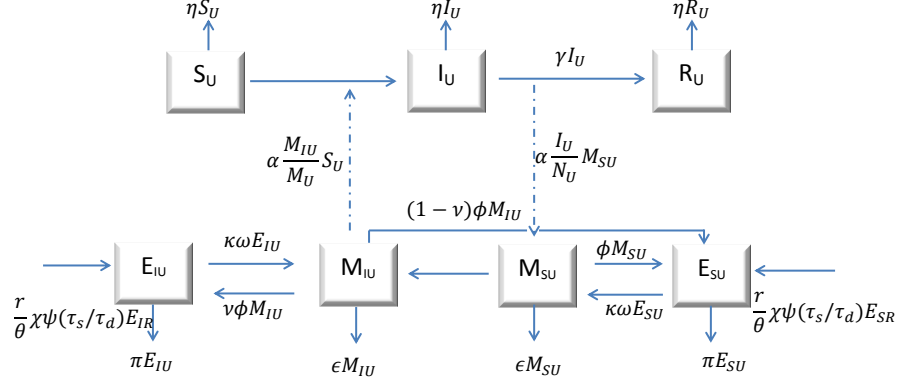


Figure 2: **Flowchart from urban dengue fever model.** Elements of the upper row refer to segments of the human population, whereas the lower row refers to adult mosquitoes (M) or their eggs (E). See Tables 1 and 2 for definitions of terms.

$$\begin{aligned}
\dot{S}_U &= \eta N_U - \alpha \frac{S_U}{N_U} M_{IU} - \eta S_U, \\
\dot{I}_U &= \alpha \frac{S_U}{N_U} M_{IU} - (\eta + \gamma) I_U, \\
\dot{R}_U &= \gamma I_U - \eta R_U, \\
\dot{M}_{SU} &= \kappa \omega E_{SU} - \alpha \frac{I_U}{N_U} M_{SU} - \epsilon M_{SU}, \\
\dot{M}_{IU} &= \kappa \omega E_{IU} + \alpha \frac{I_U}{N_U} M_{SU} - \epsilon M_{IU}, \\
\dot{E}_{SU} &= \phi M_{SU} \left(1 - \frac{E_U}{C_u}\right) + (1 - \nu) \phi M_{IU} \left(1 - \frac{E_U}{C_u}\right) - (\pi + \omega) E_{SU} + \dots \\
&\quad \frac{r}{\theta} \chi \psi \left(\frac{\tau_s}{\tau_d}\right) E_{SR}, \\
\dot{E}_{IU} &= \nu \phi M_{IU} \left(1 - \frac{E_U}{C_u}\right) - (\pi + \omega) E_{IU} + \frac{r}{\theta} \chi \psi \left(\frac{\tau_s}{\tau_d}\right) E_{IR}.
\end{aligned}$$

Where

$$N_R = S_R + I_R + R_R,$$

$$N_U = S_U + I_U + R_U,$$

$$M_R = M_{SR} + M_{IR},$$

$$E_R = E_{SR} + E_{IR},$$

$$M_U = M_{SU} + M_{IU},$$

$$E_U = E_{SU} + E_{IU}.$$

The model explicitly takes into account the movement and storage time of tires. Our study focuses on the necessary conditions for four possible outcomes. Scenario *I* considers the introduction to the urban area of mosquitoes from a rural area. In Scenario *II* a dengue outbreak emerges in the urban area as a consequence of the joint introduction of the mosquito and the virus in infected eggs. Scenario *III* induces or enhances an endemic state in the urban area through the constant introduction of infected mosquitoes eggs. Finally, in Scenario *IV*, we consider how regulation of the market in second hand tires could act as a dengue control measure. To demonstrate the impacts of dengue spread we calculate the secondary dengue cases generated in the urban area as the result of a single case in rural area. This quantity can be used as a preliminary measure of the impact of controlling the movement of tires during dengue outbreaks. Finally we apply our model to a specific study of the tire management system in Puerto Rico using data from the

Solid Waste Authority (ADS) A.D.S. (2014).

Results

Introduction of mosquitoes to an urban area

Transportation of a single batch of tires can lead to the introduction of a mosquito population from rural to urban areas. In order to obtain the conditions when this might occur, we determine the *urban offspring reproduction number* (derived in b) R_M^u by means of the next generation matrix Brauer et al. (2008). If $R_M^u > 1$, then the population of mosquitoes is able to establish itself from a small number of eggs, while if the environmental conditions cause $R_M^u < 1$ the mosquito population will eventually become extinct. In a single batch of N_T tires, the number of viable eggs that arrive in the urban area is given by

$$\frac{\omega}{\pi + \omega} \psi \left(\frac{\tau_s}{\tau_d} \right) \chi N_T \frac{E_R^*}{\theta} = \frac{\omega}{\pi + \omega} \psi \left(\frac{\tau_s}{\tau_d} \right) \chi N_T \frac{R_M^r - 1}{R_M^r} \frac{C_r}{\theta}$$

where $\frac{\omega}{\pi + \omega}$ is the probability of an egg hatching into an adult mosquito, E_R^* is the stationary number of eggs in the rural area, R_M^r is the *rural offspring reproduction number* (see b), and $\chi \psi(\frac{\tau_s}{\tau_d})$ is the fraction of eggs that survive before the tire processing cycle completes.

The introduction of a mosquito species will occur when the following conditions are met:

$$R_M^u = \frac{\kappa\omega\phi}{\epsilon(\pi + \omega)} > 1 \quad (1)$$

$$\text{and} \quad \frac{\omega}{\pi + \omega} \frac{N_T}{\theta} \chi \psi \left(\frac{\tau_s}{\tau_d} \right) \frac{R_M^r - 1}{R_M^r} C_r > 1$$

The first condition indicates that a mosquito population is sustainable in the urban area, while the second condition represents the transportation of more than one successful egg.

If tire recycling becomes an established market with a constant flux of tires from the rural to the urban area, then the expected waiting time T_M before the introduction of a mosquito species from the rural to the urban population is given by the inverse of the rate of egg introduction:

$$T_M = \left[\frac{\omega}{\pi + \omega} r \chi \psi \left(\frac{\tau_s}{\tau_d} \right) \frac{R_M^r - 1}{R_M^r} \frac{C_r}{\theta} \right]^{-1} \quad \text{for} \quad R_M^r > 1 \quad (2)$$

A dengue outbreak occurs

If continuous introduction of tires takes place from a dengue-endemic rural area to the urban area, a dengue outbreak might be precipitated by transportation of infected eggs. In order for this to happen the conditions (1) must be met, but also requires that the *basic reproductive number without vertical transmission* in the urban area $R_0^u > 1$. Its value is given by $R_0^u = \sqrt{\frac{\alpha}{\epsilon} \frac{\beta N}{(\eta + \gamma) M^*}}$ (derived in b). In this we omit vertical transmission as its effect

is negligible at the beginning of an outbreak. Thus:

$$\sqrt{\frac{\alpha}{\epsilon} \frac{\beta N}{(\eta + \gamma) M^*}} > 1 \quad (3)$$

should be met in addition to (1).

The characteristic waiting time before an outbreak T_o is given by the inverse of the effective rate of introduction of infected female mosquitoes:

$$T_o = \left[\frac{\kappa \omega}{\pi + \omega} r \chi \psi \left(\frac{\tau_s}{\tau_d} \right) \frac{E_{IR}^*}{\theta} \right]^{-1} \quad (4)$$

where $\frac{r}{\theta} \chi \psi \left(\frac{\tau_s}{\tau_d} \right)$ is the fraction of successfully imported eggs, $\frac{\kappa \omega}{\pi + \omega}$ is the probability of an egg hatching into a female mosquito before dying of natural causes.

In the case of introduction of a single batch of N_T tires, in addition to (1) and (3), the following condition must be satisfied:

$$\frac{\kappa \omega}{\pi + \omega} N_T \chi \psi \left(\frac{\tau_s}{\tau_d} \right) \frac{E_{IR}^*}{\theta} > 1 \quad (5)$$

This effectively states that more than one infected female mosquito needs to be introduced.

Endemic dengue states can be induced and enhanced

There may be situations in which $R_0^u < 1$, and therefore dengue infestation is not self-sustaining, but where continuous introduction of infected eggs in tires from an endemic rural area can induce an endemic state in the

urban area. This state is not maintained by the intrinsic dynamics of the disease in the urban area and will cease if the introduction of infected eggs is interrupted see Figs 3 and 4.

In this situation, the expected number of dengue cases at any given time I_U^* is determined as

$$I_U^* = \frac{M_{IU}^*}{M_{IU}^* - N_U \frac{\eta}{\alpha}} \quad (6)$$

where M_{IU}^* is found from the solution of the following quadratic equation

$$\begin{aligned} & (R_o^u)^2(\eta + \gamma)\left(\frac{\eta + \gamma}{\beta} + N_u\right)(M_{IU}^*)^2 \\ & - (\eta + \gamma) \left((R_o^u)^2 \left(\frac{\kappa\omega}{\beta} E_{IU}^* + N_U M_U \right) - \eta N_U \right) M_{IU}^* \\ & - \frac{\kappa\omega}{\epsilon} \eta N_U E_{IU}^* = 0 \end{aligned} \quad (7)$$

and E_{IU}^* is given by

$$E_{IU}^* = \frac{r\chi\psi\left(\frac{\tau_s}{\tau_d}\right)}{\theta(\omega + \pi)} E_{IR}^*$$

Where dengue is already endemic in the urban area, the continuous importation of tires can enhance the number of infected people *see Figs 3 and 4*. The number of infections at any given time is given by (6) when $R_0^u > 1$ in (7).

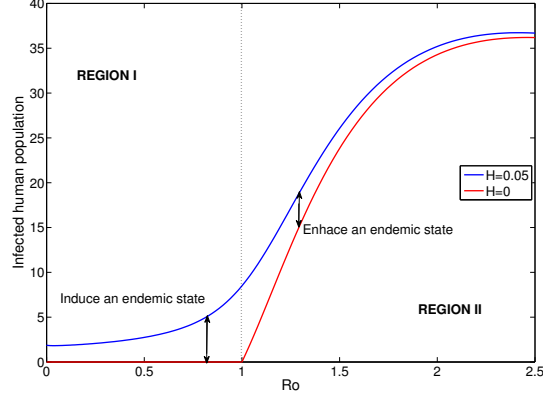


Figure 3: **Enhance an endemic state.** It is possible to induce a dengue-endemic state even though $R_0^u < 1$ (Region I) if there is a continuous flow of tires from an endemic rural area. If the disease is already endemic, tire transport of eggs will enhance the endemic state (Region II). $H = \frac{\tau}{\theta} \chi \psi \left(\frac{\tau_s}{\tau_d} \right)$ represents variation of the flow parameters of tires.

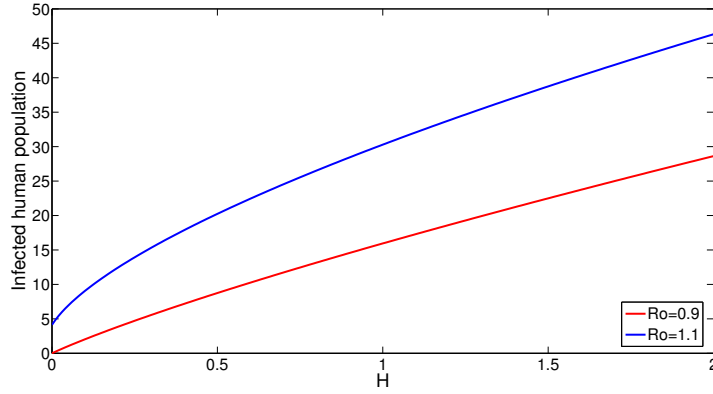


Figure 4: **Stationary level of infection in the urban population with increasing introduction of eggs.** Dengue cases increase as the number of introduced infected eggs per unit time H is increased. Red line $R_0^u < 1$ (intrinsically non-endemic state) and blue line $R_0^u > 1$ (endemic state).

Regulation of the second hand tire market as a dengue control measure

If tires are processed immediately, or at least soon after arrival to the urban area such that $\tau_s \ll \tau_d$, introduction of dengue fever does not occur.

This is because eggs die during tire processing before hatching.

In the rural area, diminishing the number of eggs reduces *rural reproduction number* (see b) R_0^r and hence the number of dengue cases. In order for this to occur, the reduction of eggs in the rural area should change the abundance of offspring from $R_M^r > 1$ to $R_M^r < 1$. If this does not happen then the risk of introducing infected eggs into the urban area will persist unless tires are processed immediately. The maximum storage time τ_s with which to minimize the risk of dengue introduction can be estimated by ensuring that the expected latency before introduction of an infected female mosquito (4) is greater than the extinction time of the vector in the rural area.

We can use the Jacobian matrix when $R_M^r < 1$ of the vector demography (B.1), to obtain the characteristic extinction time. The inverse of the smallest absolute value from its eigenvalues is an estimator of its extinction time. Thus, the following condition should be met to reduce the risk of dengue dispersal in an established market where tires are continuously imported to the urban area:

$$T_o < \left| \frac{1}{2}(\gamma + \sqrt{\xi}) \right|^{-1} \quad \text{and} \quad R_M^r = \frac{\kappa\omega\phi}{\epsilon(\pi + \omega + r/\theta)} < 1$$

where $\gamma = -(\epsilon + \pi + \omega + r/\theta)$, $\xi = \gamma^2 - 4\Xi$ and $\Xi = \epsilon(\pi + \omega + r/\theta)(1 - R_M^r)$ (see b). This simultaneously works as a control measure in the rural area.

In order to assess the impact of interventions in the tire trade on disease dynamics, we can calculate the secondary human infections in the urban

disease free area caused by human infections in the rural area at the beginning of an outbreak $R_{r \rightarrow u}$ (see b). There will be one initial case of dengue virus in the urban area related to tire transportation for each $1/R_{r \rightarrow u}$ cases in the rural area, where

$$R_{r \rightarrow u} = \frac{\alpha \kappa \omega r \chi}{\epsilon(\omega + \pi)(\theta(\omega + \pi) + r)} \psi \left(\frac{\tau_s}{\tau_d} \right) \frac{\nu \phi}{\epsilon} \frac{\beta}{(\eta + \gamma)}.$$

Thus, $R_{r \rightarrow u}$ gives the number of cases in urban area which are derived from an infected person in the rural area.

Case study

One of the main barriers for dengue eradication in Latin America is the problem of stored tires. These are a favorable site for the breeding of multiple vector insects, with implications for disease transmission and human health. As mentioned previously, used tires are one of the favored sites in which *Ae. aegypti* females deposit their eggs, becoming an important pathway for their proliferation and thus causing outbreaks of dengue in tropical and subtropical countries. An endemic disease state can arise in areas where the environmental conditions are suitable Cantanhede and Monge (Lima, 2012).

Some Latin American countries, such as Costa Rica and Peru, have banned the import of used tires. This is due to the perceived danger to public health, in addition to concerns regarding road safety and protection of the environment. Costa Rica does not possess the necessary technology to

treat used tires without causing environmental pollution Cantanhede and Monge (Lima, 2012).

In Puerto Rico the accumulation of discarded tires in *gomerías* and facilities around the island represent an environmental and health crisis. The country has therefore implemented a tire management program. According the Solid Waste Authority (ADS), around 18,000 tires are discarded every day; this amounts to 4.7 million tires a year. Despite the tire management program, it is not possible to recover all discarded tires. Among the major public health risks of excessive accumulation of tires is the spread of pests and diseases such as dengue. A total of 6,766 confirmed cases of dengue were reported in Puerto Rico in 2013. For this reason the authorities have decided to reduce the disposal of tires. They have introduced authorized solid waste facilities in which the accumulation of tires is permitted for up to 90 days. The law also allows local governments to collect used tires voluntarily and temporarily. The collection and transport of discarded tires is carried out by official vehicles A.D.S. (2014).

Fig 5 shows locations of tire storage centers in Puerto Rico. In this section we employ our model to analyze the implications of tire management for dengue transmission in this specific geographical context.

Figs 6 and 7 show the populations of infected humans in rural and urban areas. In Fig 6 the infected population in rural areas with no transportation of tires (*red line*) demonstrates that the disease is endemic. If transportation of tires takes place, the infected population in rural areas



Figure 5: The Solid Waste Authority (ADS) has detected all the centers of sale and storage of discarded tires (*gomer*as) in each state on the island. The green points show the official temporary storage centers of tires in the municipalities of Hormigueros, Toa Baja and Las Piedras). **Source:** ADS.

Parameter	Value	Units	Reference
η	0.002	1/days	Estimated
γ	1/7	1/days	Adams and Boots (2010)
α	0.67	1/days	Adams and Boots (2010)
C_a	10000 and 1000	eggs	Estimated
ϕ	10	1/days	Esteva et al. (2006)
ϵ	1/8	1/days	Adams and Boots (2010)
π	1/8	1/days	Adams and Boots (2010)
ν	0.3	proportion	Adams and Boots (2010)
ω	1/8	1/days	Adams and Boots (2010)
κ	0.5	proportion	Estimated
$\bar{\rho}$	10, 20	1/days	Estimated
χ	0.64	1/days	Estimated using A.D.S. (2014)
$\psi(\tau_s/\tau_d)$		proportion	
τ_s, τ_d	90, 10	days	A.D.S. (2014), Esteva et al. (2006)

Table 3: Parameter values for simulating the dynamics of dengue transmission in Puerto Rico based on literature sources.

declines by 9.79% (*blue solid line*) and it becomes possible for the disease to be eradicated.

In contrast, if there are around twenty infected eggs per tire and transportation to the storage sites is made systematically, the number of infected people in the rural area is reduced by 28.61% (black dash-dot line). In this situation tire transportation acts as a control measure in rural areas.

Puerto Rico has instituted a program to recover used tires. Fig (7) shows the comparison between number of infected people depending on whether the

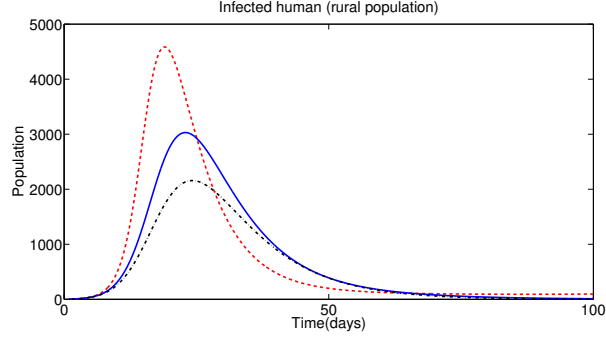


Figure 6: **Dynamics of dengue infection in rural areas under different scenarios.** Red dashed line shows the population size of infected humans when there is no transportation of tires with infected eggs. Blue solid line shows the infected population when tire transportation takes place, and black dash-dot line shows the infected population when the average of infected eggs per tire is increased

tires are handled appropriately. The blue line depicts the infected population in the absence of a recycling program. The recycling program recovers and processes 36% of tires in the three temporary storage centers. Taking this into account, the program has reduced the number of infected people in urban areas by 13.04% (*red line*).

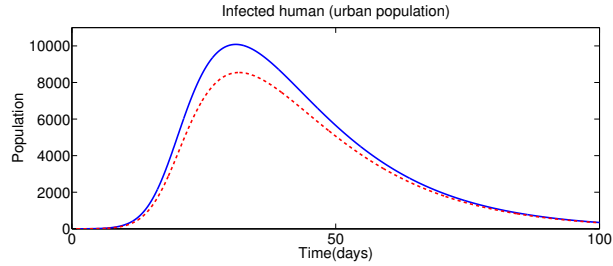


Figure 7: **Dynamics of dengue infection in urban areas, indicating the impact of the tire recycling program.** Blue solid line shows the outbreak dynamics with unregulated transport of tires, red dashed line shows the predicted urban outbreak given the existence of this program.

Discussion

The trade in second-hand tires, if unregulated, is an important factor in the generation of dengue outbreaks and can cause endemic states to be hard to eliminate. Discarded tires are believed to be one of the most productive hatcheries of the *Aedes* mosquitoes which transmit dengue fever Alves Honório et al. (2006), with many eggs being transported whilst in a diapause state. As a result of vertical transmission across generations, infected mosquitoes can pass the virus to their offspring, and therefore a new generation of infected vectors will emerge able to transmit the disease without first feeding on an infected individual. Evidence also exists of vertical transmission of Zika and Chikungunya viruses by *Ae. aegypti* and *Ae. albopictus* Thangamani et al. (2016); Ferreira-de-Brito et al. (2016); Niyas et al. (2010); Agarwal et al. (2014), extending the implications of our model to other vector-borne diseases. Our model therefore has important consequence for the spread of a range of emerging diseases. Even in hostile environmental conditions, the resistance of *Aedes* eggs to desiccation, combined with vertical transmission of the virus, is likely to facilitate the persistence and spread of disease.

Our model demonstrates that the movement of tires containing mosquito eggs has the potential to transfer both vector and virus from rural to urban regions, and with a sufficient rate of input, can induce an endemic dengue state in the urban area even if control measures would otherwise cause it to be eradicated. Management of the tire trade to reduce their storage time is a

potential strategy for reducing dispersal of the disease, and we demonstrate using an empirical case study from Puerto Rico that even a modest program of tire collection from urban areas can lead to major declines in the disease burden experienced in urban areas.

Tires left in the open are productive *Ae. aegypti* hatcheries Higa et al. (2010); Yee (2008b); Alves Honório et al. (2006), increasing the risk of dengue transmission. There is also substantial evidence that the transportation of second-hand tires between urban areas has led to the introduction or re-emergence of dengue in areas previously free of disease Belli et al. (2015); Medlock et al. (2012); Kourí et al. (1998).

Our model explicitly takes into account the movement and storage of second-hand tires, typically from rural to urban areas for processing, a common feature of their trade. In our study we demonstrate the conditions leading to four scenarios. In the first, the transport of a batch of tires to a zone free of disease introduces the vector species. This occurs when appropriate environmental conditions mean $R_M^u > 1$, and the latency before a mosquito invasion occurs can be estimated as T_m (2). Furthermore, the analysis shows that a continuous flow of tires from the endemic rural area, combined with vertical transmission of the virus, can lead to a dengue outbreak in the urban area. This happens when $R_0^u > 1$ (3). From this it is possible to estimate the critical size (5) of a batch of tires that will trigger an outbreak in the urban area. Moreover, the expected latency before an outbreak T_0 if unregulated importation of tires takes place can be estimated (4).

A third scenario is constructed when dengue is endemic in the rural area. A continuous introduction of infected eggs in tires to the urban area can induce an urban endemic state even though $R_0^u < 1$. This state is not maintained by the intrinsic dynamics of the disease in the urban area, and therefore would abate if the transportation of tires were to be interrupted. On the other hand, if the urban area is already a dengue endemic region, the introduction of tires with infected eggs serves to enhance the endemic state, that is, it increases the number of infected people.

The risk of a dengue outbreak can be reduced if an intervention is made to regulate the trade in second hand tires by setting appropriate limits on their storage time. In the ideal case, where the tires are processed immediately upon reaching the urban area $\tau_s \ll \tau_d$, an outbreak of dengue fever does not take place. In addition, removal of tires works as a control measure in rural areas by reducing the overall density of mosquito eggs and therefore in turn the number of dengue cases falls.

Diapause of mosquito eggs and vertical transmission of the virus make distinct contributions to the dynamics of disease transmission. While diapause can facilitate the spread of mosquitoes, increasing the risk of their invasion of new areas, the combination of diapause with vertical transmission can simultaneously spread the virus and cause a dengue outbreak. It is this process, mediated through the movement of reservoirs of eggs, which gives the transportation of tires such a crucial role in the spread of the disease, but also opens the opportunity for control measures. When applied to the reg-

ulated movement and storage of tires in the specific context of Puerto Rico, our model shows that even an incomplete and spatially-limited intervention can substantially reduce the disease burden experienced by both rural and urban populations. Much greater control could be achieved were tires to be reliably stored in a dry place with a limit on storage time when kept in the open. These are effective and relatively low-cost actions.

By estimating the number of secondary human infections in the urban area caused by each human infection in the rural area, we provide a means by which to evaluate the impact of both tire transportation and regulatory mechanisms. This could be used by policy makers to highlight the efficacy of interventions in a striking and accessible fashion which emphasizes the direct social implications.

Acknowledgments

Carlos Castillo-Chavez and Anuj Mubayi made helpful comments on an early version of the manuscript. We are also grateful for support from the SAL-MCMSC to participate in MTBI where part of this work was developed. We are grateful for support from CONACyT and SEP-PRODEP Grant DSA/103.5/15/7449 (Mexican agencies).

Appendix A: Urban offspring reproduction number

Considering the last one four equation of urban model and doing $\dot{M}_{SU} + \dot{M}_{IU}$ and $\dot{E}_{SU} + \dot{E}_{IU}$ it is obtained.

$$\begin{aligned}
\dot{M} &= \kappa\omega E - \epsilon M, \\
\dot{E} &= \phi M - (\pi + \omega)E + \frac{r\chi}{\theta}\psi\left(\frac{\tau_s}{\tau_d}\right)E_R.
\end{aligned} \tag{A.1}$$

We calculate the urban offspring number, using the method Brauer et al. (2008) we write the system(A.1) as $\dot{\mathfrak{X}} = \mathfrak{F} - \mathfrak{V}$.

$$\dot{\mathfrak{X}} = \begin{pmatrix} \dot{M} \\ \dot{E} \end{pmatrix}, \quad \mathfrak{F} = \begin{pmatrix} \kappa\omega E \\ 0 \end{pmatrix}, \quad \mathfrak{V} = \begin{pmatrix} \epsilon M \\ (\pi + \omega)E - \phi M \left(1 - \frac{E}{C}\right) - \frac{r\chi}{\theta}\psi\left(\frac{\tau_s}{\tau_d}\right)E_R \end{pmatrix}.$$

The Jacobian matrices F and V , associated with \mathfrak{F} and \mathfrak{V} respectively, at the vector free equilibrium $M^* = 0$, $E^* = 0$ are.

$$\begin{aligned}
F &= \begin{pmatrix} 0 & \kappa\omega \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \epsilon & 0 \\ -\phi & (\pi + \omega) \end{pmatrix}, \quad V^{-1} = \begin{pmatrix} \frac{1}{\epsilon} & 0 \\ \frac{\phi}{\epsilon(\pi + \omega)} & \frac{1}{\pi + \omega} \end{pmatrix}, \\
K = FV^{-1} &= \begin{pmatrix} \frac{\kappa\omega\phi}{\epsilon(\pi + \omega)} & \frac{\kappa\omega}{\pi + \omega} \\ 0 & 0 \end{pmatrix}.
\end{aligned}$$

The eigenvalues of K are 0 and $\frac{\kappa\omega\phi}{\epsilon(\pi+\omega)}$, so the urban offspring reproduction number is given by:

$$R_M^u = \frac{\kappa\omega\phi}{\epsilon(\pi + \omega)}$$

Appendix B: Vector demography

Considering the last one four equation of rural model and doing $\dot{M}_{SR} + \dot{M}_{IR}$ and $\dot{E}_{SR} + \dot{E}_{IR}$ it is obtained.

$$\begin{aligned}\dot{M} &= \kappa\omega E - \epsilon M, \\ \dot{E} &= \phi M - (\pi + \omega + \frac{r}{\theta})E.\end{aligned}\tag{B.1}$$

We calculate the basic offspring number of rural mosquitoes, using the method Brauer et al. (2008) we write the system(B.1) as $\dot{\mathfrak{X}} = \mathfrak{F} - \mathfrak{V}$.

$$\dot{\mathfrak{x}} = \begin{pmatrix} \dot{M} \\ \dot{E} \end{pmatrix}, \quad \mathfrak{F} = \begin{pmatrix} \kappa\omega E \\ 0 \end{pmatrix}, \quad \mathfrak{V} = \begin{pmatrix} \epsilon M \\ (\pi + \omega + \frac{r}{\theta})E - \phi M (1 - \frac{E}{C}) \end{pmatrix}.$$

The Jacobian matrices F and V , associated with \mathfrak{F} and \mathfrak{V} respectively, at the vector free equilibrium $M^* = 0, E^* = 0$ are.

$$F = \begin{pmatrix} 0 & \kappa\omega \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \epsilon & 0 \\ -\phi & (\pi + \omega + \frac{r}{\theta}) \end{pmatrix}, \quad V^{-1} = \begin{pmatrix} \frac{1}{\epsilon} & 0 \\ \frac{\phi}{\epsilon(\pi + \omega + \frac{r}{\theta})} & \frac{1}{\pi + \omega + \frac{r}{\theta}} \end{pmatrix},$$

$$K = FV^{-1} = \begin{pmatrix} \frac{\kappa\omega\phi}{\epsilon(\pi + \omega + \frac{r}{\theta})} & \frac{\kappa\omega}{\pi + \omega + \frac{r}{\theta}} \\ 0 & 0 \end{pmatrix}.$$

The eigenvalues of K are 0 and $\frac{\kappa\omega\phi}{\epsilon(\pi + \omega + \frac{r}{\theta})}$, so the rural offspring reproduction number is given by:

$$R_M^r = \frac{\kappa\omega\phi}{\epsilon(\pi + \omega + \frac{r}{\theta})}$$

The system has two stationary states $E^* = M^* = 0$ and $M^* = \frac{C\kappa\omega}{\epsilon} \left(\frac{R_M^r - 1}{R_M^r} \right)$, $E^* = C \left(\frac{R_M^r - 1}{R_M^r} \right)$.

The linearizing around the trivial stationary solutions is done. For this, we will calculate the Jacobian matrix around the equilibrium point $(0, 0)$.

$$\mathbf{DF}(\mathbf{0}, \mathbf{0}) = \begin{pmatrix} -\epsilon & \kappa\omega \\ \phi & -(\pi + \omega + \frac{r}{\theta}) \end{pmatrix}.$$

We get the following characteristic polynomial,

$$\lambda^2 + (\epsilon + \pi + \omega + \frac{r}{\theta})\lambda + \epsilon(\pi + \omega + \frac{r}{\theta})(1 - R_M) = 0$$

whose roots are of the shape

$$\lambda_{\pm} = \frac{1}{2}(\gamma \pm \sqrt{\xi})$$

where $\gamma = -(\epsilon + \pi + \omega + \frac{r}{\theta})$, $\xi = \gamma^2 - 4\Xi$ and $\Xi = \epsilon(\pi + \omega + \frac{r}{\theta})(1 - R_M)$.

Appendix C: Basic reproduction number without vertical transmission in urban area

In the following we calculate the urban reproductive number. The infected classes on urban model are: I_U , E_{IU} and M_{IU} , so the matrices \mathfrak{F} and \mathfrak{V} take the following shape.

$$\mathfrak{F} = \begin{pmatrix} \alpha \frac{S_U}{N_U} M_{IU} \\ 0 \\ \alpha \frac{I_U}{N_U} M_{SU} \end{pmatrix} \quad \mathfrak{V} = \begin{pmatrix} (\eta + \gamma) I_{IU} \\ (\pi + \omega) E_{IU} - \frac{r}{\theta} \chi \psi(\tau_s / \tau_d) E_{IR} \\ \epsilon M_{IU} - \kappa \omega E_{IU} \end{pmatrix}$$

The Jacobian matrices are:

$$F = \begin{pmatrix} 0 & 0 & \alpha \frac{S_U}{N_U} \\ 0 & 0 & 0 \\ \alpha \frac{M_{SU}}{N_U} & 0 & 0 \end{pmatrix} \quad V = \begin{pmatrix} (\eta + \gamma) & 0 & 0 \\ 0 & (\pi + \omega) & 0 \\ 0 & -\kappa\omega & \epsilon \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{(\eta + \gamma)} & 0 & 0 \\ 0 & \frac{1}{\pi + \omega} & 0 \\ 0 & \frac{\kappa\omega}{\epsilon(\pi + \omega)} & \frac{1}{\epsilon} \end{pmatrix}$$

Then we evaluated the Jacobian matrices at the disease free equilibrium $S_U = N_U, M_{SU} = M_U, E_{SU} = E_U, I_U = M_{IU} = E_{IU} = 0$. Then we found the eigenvalues of $K = FV^{-1}$ since the basic reproductive number is the spectral radius, then we need the maximum of the eigenvalues of K , this will be the basic reproductive number.

$$K = \begin{pmatrix} 0 & \frac{\alpha\kappa\omega}{\epsilon(\pi + \omega)} & \frac{\alpha}{\epsilon} \\ 0 & 0 & 0 \\ \frac{\alpha N}{(\eta + \gamma)M^*} & 0 & 0 \end{pmatrix}$$

There are three eigenvalues, one of them is zero, the other is smaller, so the maximum is

$$R_0^u = \sqrt{\frac{\alpha N}{(\mu + \gamma)M^*} \frac{\alpha}{\epsilon}}$$

Appendix D: Basic reproduction number in rural area

We calculated the basic reproductive number using the next generation matrix method $\dot{\mathfrak{X}} = \mathfrak{F} - \mathfrak{V}$ Brauer et al. (2008). The infected classes on rural model are: I_R , E_{IR} and M_{IR} , then, the information is separated into two matrices, the first one corresponds to new infection and the second one corresponds to disease progression, that is

$$\dot{\mathfrak{X}} = \begin{pmatrix} \dot{I} \\ \dot{E}_{IR} \\ \dot{M}_{IR} \end{pmatrix}$$

$$\mathfrak{F} = \begin{pmatrix} \alpha \frac{S_R}{N_R} M_{IR} \\ \nu \phi M_{IR} \left(1 - \frac{E_R}{C_r}\right) \\ \alpha \frac{I_R}{N_R} M_{SR} \end{pmatrix} \quad \mathfrak{V} = \begin{pmatrix} (\eta + \gamma) I_{IR} \\ (\pi + \omega + \frac{r}{\theta}) E_{IR} \\ \epsilon M_{IR} - \kappa \omega E_{IR} \end{pmatrix}$$

The Jacobian matrices are:

$$F = \begin{pmatrix} 0 & 0 & \alpha \frac{S_R}{N_R} \\ 0 & -\frac{\nu \phi M_{IR}}{C_r} & \nu \phi \left(1 - \frac{E_R}{C_r}\right) \\ \alpha \frac{M_{SR}}{N_R} & 0 & 0 \end{pmatrix} \quad V = \begin{pmatrix} (\eta + \gamma) & 0 & 0 \\ 0 & (\pi + \omega) + \frac{r}{\theta} & 0 \\ 0 & -\kappa \omega & \epsilon \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{(\eta+\gamma)} & 0 & 0 \\ 0 & \frac{\theta}{\theta(\pi+\omega)+r} & 0 \\ 0 & \frac{\theta\kappa\omega}{\epsilon(\theta(\pi+\omega)+r)} & \frac{1}{\epsilon} \end{pmatrix}$$

We evaluated the Jacobian matrices at the disease free equilibrium $S_R = N_R$, $M_{SR} = M_R$, $E_{SR} = E_R$, $I_R = M_{IR} = E_{IR} = 0$. Then we found the eigenvalues of $K = FV^{-1}$ since the basic reproductive number is the spectral radius. From this we need the maximum of the eigenvalues of K , which is the basic reproductive number.

$$K = \begin{pmatrix} 0 & \frac{\alpha\theta\kappa\omega}{\epsilon(\theta(\pi+\omega)+r)} & \frac{\alpha}{\epsilon} \\ 0 & \frac{\nu\phi\theta\kappa\omega}{\epsilon(\theta(\pi+\omega)+r)} \left(1 - \frac{E}{C_r}\right) & \frac{\nu\phi}{\epsilon} \left(1 - \frac{E}{C_r}\right) \\ \frac{\alpha N}{M^*\eta+\gamma} & 0 & 0 \end{pmatrix}$$

There are three eigenvalues, one of them is zero, the other is smaller, so the maximum is

$$R_0^r = \frac{1}{2} \frac{\nu\phi\kappa\omega\theta}{\epsilon(\theta(\pi+\omega)+r)} \left(1 - \frac{E_R^*}{C_r}\right) + \frac{1}{2} \sqrt{\left(\frac{\nu\phi\kappa\omega\theta}{\epsilon(\theta(\pi+\omega)+r)} \left(1 - \frac{E_R^*}{C_r}\right)\right)^2 + \frac{4\alpha}{\epsilon} \frac{\alpha N}{(\eta+\gamma)M^*}}$$

Appendix E: Basic reproduction number in urban area

We calculate the urban reproductive number. The infected classes in the urban model are: I_U , E_{IU} and M_{IU} , so the matrices \mathfrak{F} and \mathfrak{V} take the following shape:

$$\mathfrak{F} = \begin{pmatrix} \alpha \frac{S_U}{N_U} M_{IU} \\ \nu \phi M_{IU} \left(1 - \frac{E_U}{C_u}\right) \\ \alpha \frac{I_U}{N_U} M_{SU} \end{pmatrix} \quad \mathfrak{V} = \begin{pmatrix} (\eta + \gamma) I_{IU} \\ (\pi + \omega) E_{IU} - \frac{r}{\theta} \chi \psi(\tau_s/\tau_d) E_{IR} \\ \epsilon M_{IU} - \kappa \omega E_{IU} \end{pmatrix}$$

The Jacobian matrices are:

$$F = \begin{pmatrix} 0 & 0 & \alpha \frac{S_U}{N_U} \\ 0 & -\frac{\nu \phi M_{IU}}{C_u} & \nu \phi \left(1 - \frac{E_U}{C_u}\right) \\ \alpha \frac{M_{SU}}{N_U} & 0 & 0 \end{pmatrix} \quad V = \begin{pmatrix} (\eta + \gamma) & 0 & 0 \\ 0 & (\pi + \omega) & 0 \\ 0 & -\kappa \omega & \epsilon \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{(\eta + \gamma)} & 0 & 0 \\ 0 & \frac{1}{\pi + \omega} & 0 \\ 0 & \frac{\kappa \omega}{\epsilon(\pi + \omega)} & \frac{1}{\epsilon} \end{pmatrix}$$

Then we evaluated the Jacobian matrices at the disease free equilibrium $S_U = N_U$, $M_{SU} = M_U$, $E_{SU} = E_U$, $I_U = M_{IU} = E_{IU} = 0$. We found the

eigenvalues of $K = FV^{-1}$ since the basic reproductive number is the spectral radius. The maximum of the eigenvalues of K is the basic reproductive number.

$$K = \begin{pmatrix} 0 & \frac{\alpha\kappa\omega}{\epsilon(\pi+\omega)} & \frac{\alpha}{\epsilon} \\ 0 & \frac{\nu\phi\kappa\omega}{\epsilon(\pi+\omega)} \left(1 - \frac{E_U}{C_u}\right) & \frac{\nu\phi}{\epsilon} \left(1 - \frac{E_U}{C_u}\right) \\ \frac{\alpha N}{(\eta+\gamma)M^*} & 0 & 0 \end{pmatrix}$$

There are three eigenvalues, one of them is zero, the other is smaller, so the maximum is

$$R_0^u = \frac{1}{2} \frac{\nu\phi\kappa\omega}{\epsilon(\pi+\omega)} \left(1 - \frac{E_U^*}{C_u}\right) + \frac{1}{2} \sqrt{\left(\frac{\nu\phi\kappa\omega}{\epsilon(\pi+\omega)} \left(1 - \frac{E_U^*}{C_u}\right)\right)^2 + \frac{4\alpha}{\epsilon} \frac{\alpha N}{(\eta+\gamma)M^*}}$$

The basic reproductive number of the complete model is the maximum of the two reproductive numbers, *rural reproduction number* and *urban reproduction number*.

$$R_0 = \max \{R_0^r, R_0^u\}$$

Appendix F: Number of transmissions from rural to urban area

To find transmission from rural to urban reproduction number, we following the same idea to calculate the above basic reproduction numbers, so we want to know how many infections could cause an individual of the rural population in the urban population by the movement of infected tires, for that we are consider that in rural population the disease is endemic. The infected classes on full model are: I_R , E_{IR} , M_{IR} , I_U , E_{IU} , M_{IU} . The information is separated into two matrices, the first one corresponds to new infection \mathfrak{F} and the second one corresponds to disease progression \mathfrak{V} , that is

$$\mathfrak{F} = \begin{pmatrix} \alpha \frac{S_U}{N_U} M_{IU} \\ \nu \phi M_{IU} \left(1 - \frac{E_U}{C_u}\right) \\ \alpha \frac{I_U}{N_U} M_{SU} \\ \alpha \frac{S_R}{M_R} M_{IR} \\ \nu \phi M_{IR} \left(1 - \frac{E_R}{C_r}\right) \\ \alpha \frac{I_R}{N_R} M_{SR} \end{pmatrix} \quad \mathfrak{V} = \begin{pmatrix} (\eta + \gamma) I_{IU} \\ (\pi + \omega) E_{IU} - \frac{r}{\theta} \chi \psi(\tau_s/\tau_d) E_{IR} \\ \epsilon M_{IU} - \kappa \omega E_{IU} \\ (\eta + \gamma) I_{IR} \\ (\pi + \omega) E_{IR} + \frac{r}{\theta} E_{IR} \\ \epsilon M_{IR} - \kappa \omega E_{IR} \end{pmatrix}$$

The Jacobian matrices are:

$$F = \begin{pmatrix} 0 & \alpha \frac{S_U}{N_U} & 0 & 0 & 0 & 0 \\ \alpha \frac{M_{SU}}{N_U} & 0 & 0 & 0 & 0 & 0 \\ 0 & \nu \phi \left(1 - \frac{E_U}{C_u}\right) & -\frac{\nu \phi M_{IU}}{C_u} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha \frac{S_R}{N_R} & 0 \\ 0 & 0 & 0 & \alpha \frac{M_{SR}}{N_R} & 0 & 0 \\ 0 & 0 & 0 & 0 & \nu \phi \left(1 - \frac{E_R}{C_r}\right) & -\frac{\nu \phi M_{IR}}{C_r} \end{pmatrix}$$

$$V = \begin{pmatrix} (\eta + \gamma) & 0 & 0 & 0 & 0 & 0 \\ 0 & \epsilon & -\kappa \omega & 0 & 0 & 0 \\ 0 & 0 & (\pi + \omega) & 0 & 0 & -\frac{r}{\theta} \chi \psi(\tau_s/\tau_d) \\ 0 & 0 & 0 & (\eta + \gamma) & 0 & 0 \\ 0 & 0 & 0 & 0 & \epsilon & -\kappa \omega \\ 0 & 0 & 0 & 0 & 0 & (\pi + \omega + \frac{r}{\theta}) \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{(\eta+\gamma)} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\epsilon} & \frac{\kappa\omega}{\epsilon(\pi+\omega)} & 0 & 0 & \frac{\kappa\omega r\chi}{\epsilon(\pi+\omega)(\theta(\pi+\omega)+r)}\psi\left(\frac{\tau_s}{\tau_d}\right) \\ 0 & 0 & \frac{1}{(\pi+\omega)} & 0 & 0 & \frac{r\chi}{(\pi+\omega)(\theta(\pi+\omega)+r)}\psi\left(\frac{\tau_s}{\tau_d}\right) \\ 0 & 0 & 0 & \frac{1}{(\eta+\gamma)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\epsilon} & \frac{\kappa\omega\theta}{\epsilon(\theta(\pi+\omega)+r)} \\ 0 & 0 & 0 & 0 & 0 & \frac{\theta}{\theta(\pi+\omega)+r} \end{pmatrix}$$

Then we evaluated the Jacobian matrices at the disease free equilibrium. Then we found $K = FV^{-1}$. To get the *number of transmissions from rural to urban area* we obtain K^3 with this matrix in the column if infectious rural population we get the following.

$$R_{r \rightarrow u} = \frac{\alpha\kappa\omega r\chi}{\epsilon(\omega + \pi)(\theta(\omega + \pi) + r)}\psi\left(\frac{\tau_s}{\tau_d}\right) \frac{\nu\phi}{\epsilon} \frac{\alpha}{(\eta + \gamma)}$$

Appendix G: Summary of reproduction number

- Urban offspring reproduction number.

$$R_M^u = \frac{\kappa\omega\phi}{\epsilon(\pi + \omega)}$$

- Rural offspring reproduction number.

$$R_M^r = \frac{\kappa\omega\phi}{\epsilon(\pi + \omega + \frac{r}{\theta})}$$

- Basic reproduction number without vertical transmission in urban area.

$$R_0^u = \sqrt{\frac{\alpha N}{(\mu + \gamma)M^*} \frac{\alpha}{\epsilon}}$$

- Basic reproduction number in urban area.

$$R_0^u = \frac{1}{2} \frac{\nu\phi\kappa\omega}{\epsilon(\pi + \omega)} \left(1 - \frac{E_U^*}{C_u}\right) + \frac{1}{2} \sqrt{\left(\frac{\nu\phi\kappa\omega}{\epsilon(\pi + \omega)} \left(1 - \frac{E_U^*}{C_u}\right)\right)^2 + \frac{4\alpha}{\epsilon} \frac{\alpha N}{(\eta + \gamma)M^*}}$$

- Basic reproduction number in rural area.

$$R_0^r = \frac{1}{2} \frac{\nu\phi\kappa\omega\theta}{\epsilon(\theta(\pi + \omega) + r)} \left(1 - \frac{E_R^*}{C_r}\right) + \frac{1}{2} \sqrt{\left(\frac{\nu\phi\kappa\omega\theta}{\epsilon(\theta(\pi + \omega) + r)} \left(1 - \frac{E_R^*}{C_r}\right)\right)^2 + \frac{4\alpha}{\epsilon} \frac{\alpha N}{(\eta + \gamma)M^*}}$$

- Number of transmissions from rural to urban area.

$$R_{r \rightarrow u} = \frac{\alpha\kappa\omega r\chi}{\epsilon(\omega + \pi)(\theta(\omega + \pi) + r)} \psi\left(\frac{\tau_s}{\tau_d}\right) \frac{\nu\phi}{\epsilon} \frac{\alpha}{(\eta + \gamma)}$$

References

- Adams, B., Boots, M., 2010. How important is vertical transmission in mosquitoes for the persistence of dengue? Insights from a mathematical model. *Epidemics* 2 (1).
- A.D.S., 2014. Autoridad de Desperdicios Sólidos. Manejo de Neumáticos en Puerto Rico. <http://www.ads.pr.gov/programas/neumaticos/>.
- Agarwal, A., Dash, P., Singh, A., Sharma, S., Gopalan, N., Rao, P., Parida, M., Reiter, P., 2014. Evidence of experimental vertical transmission of emerging novel ECSA genotype of chikungunya virus in *aedes aegypti*. *Plos Neglect Trop D* 8 (7).
- Alves Honório, N., Cabello, P. H., et.al., 2006. Preliminary data on the performance of *aedes aegypti* and *aedes albopictus* immatures developing in water-filled tires in rio de janeiro. *Memórias do Instituto Oswaldo Cruz* 101 (2), 225–228.
- Belli, A., Arostegui, J., Garcia, J., Aguilar, C., Lugo, E., Lopez, D., Valle, S., Lopez, M., Harris, E., Coloma, J., 2015. Introduction and establishment of *aedes albopictus*(diptera: Culicidae) in managua, nicaragua. *Journal of Medical Entomology* 52 (4).
- Brauer, F., Van den Driessche, P., Wu, J., 2008. *Mathematical Epidemiology*. Vol. 408. Springer Verlag.

- Briseño-García, B., Gómez-Dantés, H., Argott-Ramírez, E., Montesano, R., Vázquez-Martínez, A. L., Ibáñez-Bernal, S., Madrigal-Ayala, G., Ruíz-Matus, C., Flisser, A., Tapia-Conyer, R., 1996. Potential risk for dengue hemorrhagic fever: the isolation of serotype dengue-3 in Mexico. *Emerging Infectious Diseases* 2 (2).
- Cantanhede, A., Monge, G., Lima, 2012. Estado del arte del manejo de llantas usadas en las Américas. Organización Panamericana de la Salud.
- Cook, G., Zumla, A., 2008. *Tropical Disease*. Twenty second edition, Saunderson Elsevier.
- Esteva, L., Rivas, G., Yang, H., 2006. Modelling parasitism and predation of mosquitoes by water mites. *Journal of Mathematical Biology* 53 (4), 540–555.
- Esteva, L., Vargas, C., 2000. Influence of vertical and mechanical transmission on the dynamics of dengue disease. *Mathematical Biosciences* 167 (1), 51–64.
- Esteva, L., Vargas, C., 2003. Coexistence of different serotypes of dengue virus. *Journal of Mathematical Biology* 46 (1), 31–47.
- Ferreira-de-Brito, A., Ribeiro, I. P., de Miranda, R., Fernandes, R., Campos, S., da Silva, K., de Castro, M., Bonaldo, M. C., Brasil, P., Ricardo, L., 2016. First detection of natural infection of *Aedes aegypti* with Zika virus

- in brazil and throughout south america. Memórias Instituto Oswaldo Cruz 111 (10), 655–658.
- García-Rivera, E. J., Rigau-Pérez, J. G., 2006. Dengue virus, Chapter 17. Humana Press.
- Gubler, D. J., 1986. The arbovirus: Epidemiology and ecology. T.P. Monath (Ed). CRC, Boca Raton, FL 2, 213.
- Gubler, D. J., 2002. Epidemic dengue/dengue hemorrhagic fever as a public health, social and economic problem in the 21st century. Trends in Microbiology 10 (2).
- Günther, J., Martínez-Muñoz, J., Pérez-Ishiwara, D., Salas-Benito, J., 2007. Evidence of vertical transmission of dengue virus in two endemic localities in the state of Oaxaca, Mexico. Intervirology 50 (5), 347–352.
- Higa, Y., Thi Yen, N., Kawada, H., Hai Son, T., Thuy Hoa, N., Takagi, M., 2010. Geographic Distribution of *Aedes aegypti* and *Aedes albopictus* Collected from Used Tires in Vietnam 26.
- Kourí, G., Guzmán, M. G., Valdés, L., Carbonel, I., del Rosario, D., Vazquez, S., Laferté, J., Delgado, J., Cabrera, M. V., 1998. Reemergence of dengue in Cuba: a 1997 epidemic in Santiago de Cuba. Emerging Infectious Diseases 4 (1).
- Martins, V. E. P., Alencar, C. H., Kamimura, M. T., de Carvalho Araújo, F. M., De Simone, S. G., Dutra, R. F., Guedes, M. I. F., 2012. Occurrence

- of Natural Vertical Transmission of Dengue-2 and Dengue-3 Viruses in *Aedes aegypti* and *Aedes albopictus* in Fortaleza, Ceará, Brazil 7.
- Medlock, J. M., Hansford, K. M., Schaffner, F., Versteirt, V., Hendrickx, G., Zeller, H., Bortel, W. V., 2012. A Review of the Invasive Mosquitoes in Europe: Ecology, Public Health Risks, and Control Options 12.
- Murillo, D., Holechek, S. A., Murillo, A. L., Sanchez, F., Castillo-Chavez, C., 2014. Vertical Transmission in a Two-Strain Model of Dengue Fever 1.
- Niyas, K. P., Abraham, R., Unnikrishnan, R., Mathew, T., Nair, S., Manakkadan, A., Issac, A., Sreekumar, E., 2010. Molecular characterization of chikungunya virus isolates from clinical samples and adult *aedes albopictus* mosquitoes emerged from larvae from kerala, south india. *Virol J* 7 (1), 1–8.
- Rezza, G., 2012. *Aedes albopictus* and the reemergence of dengue. *BMC., Public health* 12 (72).
- Thangamani, S., Huang, J., Hart, C., Guzman, H., Tesh, R., 2016. Vertical transmission of zika virus in *aedes aegypti* mosquitoes. *Am J Trop Med Hyg.*
- Thomas, S. M., Obermayr, U., et.al, 2012. Low-temperature threshold for egg survival of a post-diapause and non-diapause european aedine strain, *aedes albopictus* (diptera: Culicidae). *Parasites and vectors* 5 (100).

- World Health Organization, 2009. Dengue: Guidelines for diagnosis, treatment, prevention and control. For research on disease of poverty.
- Yee, D., 2008a. Tires as habitats for mosquitoes: a review of studies within the eastern united states. *Journal Medical Entomology* 45 (4), 81–93.
- Yee, D. A., 2008b. Tires as habitats for mosquitoes: A review of studies within the eastern united states. *Journal of Medical Entomology* 51 (4).